Секция «Вещественный, комплексный и функциональный анализ»

Boundedness of the multi-dimentional Riemann-Liouville fractional integral operator in weighted Morrey spaces

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A real-valued function f belongs to the weighted Morrey spaces $M^{\lambda}_{p,\Omega,w(\cdot)}$ if f is Lebesgue measurable on Ω and the following norm is finite

$$\|f\|_{M^{\lambda}_{p,\Omega,w(\cdot)}} = \sup_{x\in\Omega, r>0}\left(\frac{1}{r^{n\lambda}}\int_{B(x,r)\cap\Omega}|w(y)f(y)|^p dy\right)^{\frac{1}{p}},$$

where $1 \le p < \infty$, $0 \le \lambda \le 1$, $\Omega \subset \mathbb{R}^n$ is a Lebesgue measurable set, and w denotes a non-negative Lebesgue measurable function on Ω.

We consider $\Omega = \prod\limits_{i=1}^{n}(a_i, b_i), -\infty < a_i < b_i < \infty, i = 1, ..., n$, and the left multidimensional fractional Riemann-Liouville integral operator of order $\alpha = (\alpha_1, ..., \alpha_n), \alpha_i > 0, i = 1, ..., n$

$$\left(I^{\alpha}_{a+,w}f\right)(x) = \frac{1}{\prod\limits_{i=1}^{n}\Gamma(\alpha_i)}\int_{a_1}^{x_1}\cdots\int_{a_n}^{x_n}\prod_{i=1}^{n}(x_i - t_i)^{\alpha_i - 1}w(t_1, ..., t_n)f(t_1, ...t_n)dt_1...dt_n, x \in \Omega,$$

where Γ is the Euler Gamma-function.

In the following theorem conditions are given ensuring the boundedness of the multi-dimensional Riemann-Liouville fractional integral operators in weighted Morrey spaces.

Theorem. *Let* $-\infty < a_i < b_i < \infty, i = 1, ..., n, \Omega = \prod\limits_{i=1}^{n}(a_i, b_i), 1 \le p, q < \infty$, $\frac{1}{p} < \alpha_i < 1$, $i = 1, ..., n$, $0 \le \beta, \gamma \le 1$, *and* w *be a non-negative Lebesgue measurable function.*

Then the operator $I^{\alpha}_{a+,w}$ *is bounded from* $M^{\beta}_{p,\Omega,w(\cdot)}$ *to* $M^{\gamma}_{q,\Omega,w(\cdot)}$.

In the one-dimensional case and $w(x) \equiv 1$ this statement was proved in [1].

References

[1] Zun Wei Fu, Juem Trujillo, Qing Yan Wu, Riemann-Liouville Fractional Calculus in Morrey Spaces and Applications, Elsevier, 2016. Computers and Mathematics with Applications (2016), http://dx.doi.org/10.1016/j.camwa.2016.04.013